

THE APPLICABILITY OF  
NONLINEAR SYSTEMS DYNAMICS  
CHAOS MEASURES TO  
CARDIOVASCULAR PHYSIOLOGY VARIABLES

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# ABSTRACT

Three measures of nonlinear chaos--fractal dimension, Approximate Entropy (ApEn), and Lyapunov exponents--were investigated as potential measures of cardiovascular condition.

It is suggested that these measures have potential in the assessment of cardiovascular condition in environments of normal cardiovascular stress (normal gravity on the earth surface), cardiovascular deconditioning (microgravity of space), and increased cardiovascular stress (lower body negative pressure (LBNP) treatments).

## INTRODUCTION

Many cardiovascular variables, such as heart rate and blood pressure, when recorded over long periods of time, appear to be random variables. Using statistics, such as the mean and the standard deviation, and measures of confidence in these statistics, statistical analyses of stochastic systems producing true random variables, are well understood and extensively used. The cardiovascular system, however, is not a stochastic system producing true random variables, but, instead, is a highly developed, very complicated, nonlinear, dynamic system producing tightly regulated output variables.

Recently, several techniques have been developed for the analysis of the behavior of deterministic nonlinear dynamic systems. These systems produce chaotic output variables that appear to be random functions of time, but which are actually completely determined by the deterministic, though nonlinear, differential equations describing the system. Even though a deterministic description of the cardiovascular system is not known, it seems appropriate to apply these techniques to its analysis.

This report presents some of the findings of a preliminary study of the applicability of three measures of nonlinear chaos to the assessment of cardiovascular condition based on recordings of cardiovascular physiology variables. These measures are fractal dimension, Lyapunov exponents, and Approximate Entropy (ApEn).

## NONLINEAR CHAOS MEASURES

The three measures considered here are attempts at the quantification of the variability, or, inversely, the "patternness", of chaotic behavior. Two of these, fractal dimension and Lyapunov exponents, are used extensively in the characterization of deterministic nonlinear dynamic systems and have been used to a lesser extent in the attempt to describe the behavior of chaotic systems with unknown descriptions. The third, ApEn, has been developed for the analysis of heart rate data and, specifically, used in the analysis of fetal heart rate. A short discussion of each of these measures follows.

## Fractal Dimension

Fractal dimension is a means of classifying structures that exhibit self-similarity. A self-similar structure can be described as follows. If a section of a self-similar structure is viewed in magnification, the smaller-scale section is similar in form to its larger-scale parent. As an example, consider a straight line segment which is broken into smaller segments with gaps between each segment. Then each segment is again broken into smaller segments with gaps between each of these segments, etc.. If the ratio of segment length to gap length is maintained, a self-similar structure is generated, and as each segment is viewed in magnification the same form is apparent.

For self-similar structures the fractal dimension can be thought of as a measure of how the structure fills the space of the original structure. Generally, the self-similar structure will have a fractal dimension less than the dimension of the original structure. A continuous line segment has a dimension of one, while a self-similar line structure as described above will have a fractal dimension less than one.

Consider the following larger dimensioned structures. A filled square has a dimension of two, since it completely fills a two-dimensional space. A self-similar square only partially filled with smaller self-similar squares will have a fractal dimension less than two, since the blank spaces between the squares are unfilled. Similarly, a solid cube has a dimension of three, since it completely fills a three dimensional space. A self-similar cube only partially filled with smaller self-similar cubes will have a fractal dimension less than three, since spaces between the cubes are unfilled.

For self-similar structures the fractal dimension is a function of the logarithms of the number of repeated structures and the dimension of the original structure.

Self-similar structures can be much more complicated than the examples given above. Line segments can branch into the two-dimensional plane creating a fractal dimension between one and two. Other three dimensional structures such as spheres may be nested to created fractal dimensions between two and three.

Others (1) have illustrated the appearance of self-similarity in heart rate recordings of several minutes duration. Normant and Tricot (2) have suggested an approach

to the evaluation of fractal dimension of experimentally derived curves. Their method is presently under investigation in the Cardiovascular Laboratory. It may be used to determine the fractal dimension of heart rate data and other physiological variables of interest to this lab.

#### Lyapunov Exponents

Lyapunov exponents are used to obtain a measure of the sensitivity of a nonlinear dynamic system to its initial conditions. Nonlinear dynamic systems are often studied in the phase space in which trajectories appear. In the case of chaotic systems these trajectories nearly repeat themselves. Lyapunov exponents are also a measure of the average rate of divergence of these trajectories. As the order of a system increases the number of Lyapunov exponents necessary to characterize the system increases.

For deterministic systems Lyapunov exponents are determined by taking the limit of the sum of the logarithms of the derivatives of the iterations of the trajectories through the phase space. This is a complicated procedure for deterministic systems, and much more difficult for systems with unknown dynamics such as the cardiovascular system. In addition, since the order of the cardiovascular system is also unknown, the number of Lyapunov exponents necessary to describe it is also unknown. However, Wolf, et.al. (3) have suggested a method of determining Lyapunov exponents from experimental time series. Their method is also under study in the Cardiovascular Laboratory.

#### Approximate Entropy (ApEn)

Pincus (4) has proposed another measure in the ApEn. ApEn is a measure of the deviation of a time series away from a pattern or somewhat regularity. It is a measure of the increased information available in the time series with increased variability. The algorithmic computation of ApEn is similar to the computation of Lyapunov exponents but much simpler. It involves the summation of the logarithms of the number of deviations of different magnitudes. Pincus has evaluated the level of fetal distress by determining the ApEn of fetal heart rate (4). This approach is also under consideration in the Cardiovascular Laboratory.

## COMMENTS AND RECOMMENDATIONS

This preliminary study indicates that a more detailed analysis of nonlinear chaos measures applied to cardiovascular physiology variables should be vigorously pursued. There is the potential for the development of a very useful tool in the evaluation of cardiovascular condition in these measures. The analysis of chaotic systems must make use of measures other than the traditional statistics.

Present analyses indicate that these nonlinear chaos measures require time series on the order of 1000 data points in length for good resolution. For baseline data gathering situations time series of this length are easily obtained, but, for active tests such as LBNP ramp studies conducted both in the lab and in space, such time periods may not be practical. Techniques using shorter time intervals must be developed.

The following recommendations are offered:

- 1) The continued development of an analysis capability to be used in the evaluation of the three measures using Cardiovascular Laboratory data.
- 2) Using relatively long term time series of heart rate data evaluated the three measures and compare with spectral analyses of the same data. To do this and associated correlation studies an improved FFT should be incorporated.
- 3) Investigate the usefulness of incorporating other cardiovascular variables, such as blood pressure, into the analyses. This could have the effect of changing from a single function of time to a vector function of time, and possibly reduce the long time series requirement.
- 4) Consider variations of the nonlinear chaos measures to facilitate the use of shorter time series. This is of significant importance in the analysis of active LBNP ramp tests.

## REFERENCES

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